DATA 468 Homework 1

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Homework Date: March 23, 2025

Submission Date: March 30, 2025 (11:00 PM) (Beijing Time)

Submission Date in Gradescope: March 30, 2025 (8:00 AM) (Tucson Time)

**Instructions**: Please write or type your solutions clearly and show all relevant steps. Once you are done, please upload your solutions to Gradescope. Ifyou need to scan your solutions, please use a free scanning app like CamScanner instead of sendingphotographs. Please submit your solutions within the prescribed time, as late submissions will be notconsidered.

1. A probability space is defined as a triple (*Ω, F, P*). Given the following:

* The sample space Ω={H,T} , where H represents heads and T represents tails.
* The σ-algebra is ***F***={∅， {H}，{T}，{H，T}}, a collection of subsets ofΩ.
* P is a probability measure such that and P{H}=p， P{T}=1-p.

1. Show that ***F*** satisfies the properties of σ-algebra.

**Solution**: We are to show that the collection of all **events** (subsets of Ω—i.e., the **power set** of Ω—forms a **σ-algebra**.

Step 1: Define F

Since Ω={H,T}, the **power set** F is:

F={∅,{H},{T},{H,T}}

Step 2: Verify σ-algebra Properties

A collection F is a **σ-algebra** if it satisfies these three properties:

#### (i) Ω∈F

We have {H,T}=Ω∈F, so this holds.

#### (ii) If A∈FA , then Ac∈F

We check complements:

* ∅c=Ω∈F
* {H}c={T}∈F
* {T}c={H}∈F
* Ωc=∅∈F

So this property also holds.

#### (iii) Closed under countable unions

We check that any **countable union** of sets in F is also in F. Since F has only 4 elements, any countable union will still be a subset of Ω, and hence in F.

Examples:

* {H}∪{T}={H,T}∈F
* ∅∪{H}={H}∈F
* Infinite union of {H},{H},{H},⋯={H}∈F

So this holds.

### Conclusion

The set F={∅,{H},{T},{H,T}} **does satisfy all the properties of a σ-algebra** over Ω={H,T}.

(b) Verify that *P* satisfies the axioms of a probability measure.

(c) If p=0.5, compute *P*({H,T}) and *P*(∅).

Note: ∅(empty set)